

Modeling Planar Unconstrained Multi Degree of Freedom Systems

Equations of Motion

Newton's Second Law

$$\Sigma \vec{F} = \frac{d\vec{p}}{dt}$$

$\Sigma \vec{F}$: sum of forces acting on the collection of particles and rigid bodies that make up the system

\vec{p} : linear momentum

Euler's Second Law

$$\Sigma \vec{T} = \frac{d\vec{h}}{dt}$$

$\Sigma \vec{T}$: sum of torques acting on the rigid body

\vec{h} : angular momentum

These are 2nd order ordinary differential equations.
 ↑
 function of only time

For example:

$$\Sigma \vec{F} = \frac{d\vec{p}}{dt} = \frac{dm\vec{v}}{dt} \Rightarrow \int \Sigma \vec{F} dt = \int m d\vec{v}$$

$$\int \Sigma \vec{F} dt = m\vec{v} \Rightarrow \boxed{\begin{aligned} \vec{v} &= \frac{1}{m} \int \Sigma \vec{F} dt \\ \vec{x} &= \int \vec{v} dt \end{aligned}}$$

two 1st order diff. eqs.

Basic Procedure For Modeling & Analysis

- ① Create a model of reality that describe the momentum and forces acting on some system
- ② Derive the equations of motion
- ③ Seek solutions to the equations of motion
- ④ Interpret results

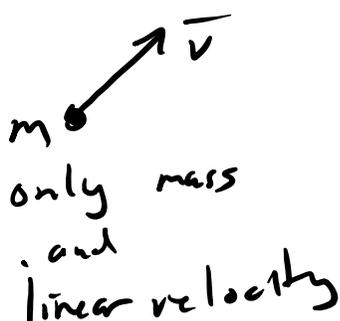
Essential Elements of Vibrating System

- ① Inertia of oscillating masses (stores kinetic energy)
- ② Restoring forces: elastic, gravity, etc. (store potential energy)
- ③ Dissipative mechanisms (energy loss)

Concepts

- system: collection of inertial elements and the forces that act on them (lumped-mass systems)

- inertial elements: particles and rigid bodies



only mass
and
linear velocity



distribution
of mass

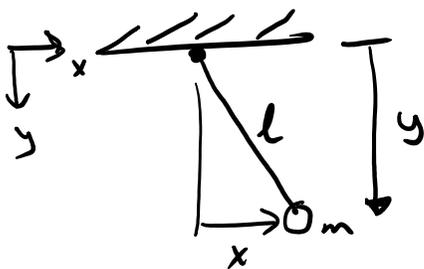
linear velocity
+ mass
rotational velocity
& inertia

and linear velocity

rotational velocity & inertia

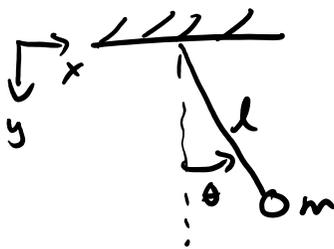
- reference frames: conceptual frame of reference to view/observe motion. most important is inertial reference frame, inertial frame has negligible acceleration wrt the observed motion. Newton's law only valid in an inertial reference frame
- Coordinate systems: reference axes fixed in a reference frame with which measure coordinates

- coordinates: positions and angles that describe a system's configuration (position and angles of a collection of rigid bodies and particles at any given time)

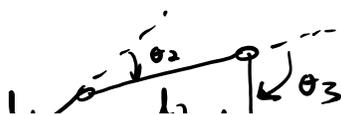


x and y are coordinates

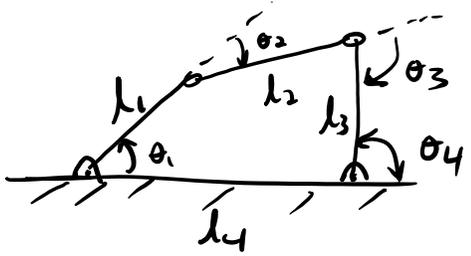
- generalized coordinates: a minimal set of coordinates that describe a system's configuration.



Single coordinate θ fully describes the configuration, thus θ is a generalized coordinate



a generalized coordinate



- generalized speeds: functions of the generalized coordinates and the time derivatives of the generalized coordinates.

$$x = l \sin \theta$$

$$v_x \triangleq \dot{x} = l \dot{\theta} \cos \theta$$

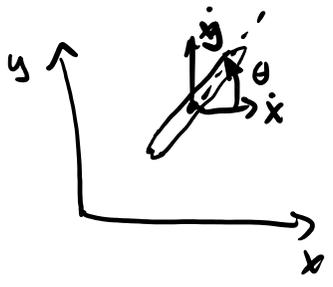
↳ example of a GS

For this class we always choose the simplest GS.

$$\omega \triangleq \dot{\theta}$$

↳ generalized speed

- degrees of freedom: for ^{motion} unconstrained systems, the # of DoF is the minimum number of coordinates (i.e. # of GCs)



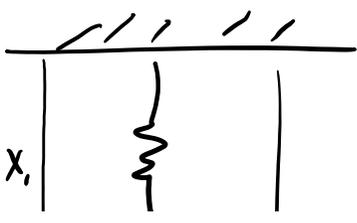
for constrained systems:

$$\# \text{ DoF} = \# \text{ GCs} - \# \text{ motion constraint equations}$$

$$\tan \theta = \frac{y}{x}$$

\$\dot{y} = \dot{x} \tan \theta \rightarrow\$ motion constraint

2 GCs



only 1 GC

