

$$d = 2r \sin\left(\frac{\theta(t)}{2}\right)$$

$$h = l - \sqrt{l^2 - d^2}$$

$$v = \frac{dh}{dt}$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \omega} \right) = \frac{\partial L}{\partial \theta} \quad \text{where } L = T - U$$

$$T = \frac{1}{2} (m_b + m_c) v^2 + \frac{1}{2} (I_b + I_c) \omega^2$$

$$U = (m_b + m_c) g h$$

$$f = \frac{d}{dt} \left(\frac{\partial L}{\partial \omega} \right) - \frac{\partial L}{\partial \theta} = 0$$

$$f = I_{sys} \dot{\omega} + g(\omega, \theta, t) = 0$$

EoM will always be of this form (with linear coeff of time derivative of G.S.)

$$\dot{\omega} = \frac{-g(\omega, \theta, t)}{I_{sys}}$$

$$\dot{\theta} = \omega$$

two first order explicit non-linear ordinary differential equation

(form required by numerical ODE solvers)

Think of linear 2nd order ODE:

$$m\ddot{x} + b\dot{x} + kx = 0$$

If $v = \dot{x}$ is introduced then:

$$\rightarrow \begin{cases} m\dot{v} + bv - kv = 0 \\ \dot{x} = v \end{cases} \text{ two 1st order ODEs!}$$

If $v = \dot{x}$ is introduced

$$m\dot{v} + bv + kx = 0 \Rightarrow \dot{v} = \frac{-bv - kx}{m}$$
$$\dot{x} = v$$

} two 1st order ODEs!
equivalent to the single second order ODE.

After linearizing f we get:

$$(I_b + I_c) \ddot{\theta} + \frac{gr(m_b + m_c)}{l} \theta = 0$$

This is the equation for an undamped system!

Like $m\dot{v} + kv = 0$ or $\dot{v} + \frac{k}{m}v = 0 \Rightarrow \dot{v} + \omega_n^2 v = 0$

So the natural frequency of our system is

$$\omega_n^2 = \frac{gr(m_b + m_c)}{l(I_b + I_c)} !$$

→ small angle (G.C.) and small motion assumptions (G.S.)

$$\sin \theta \approx \theta$$

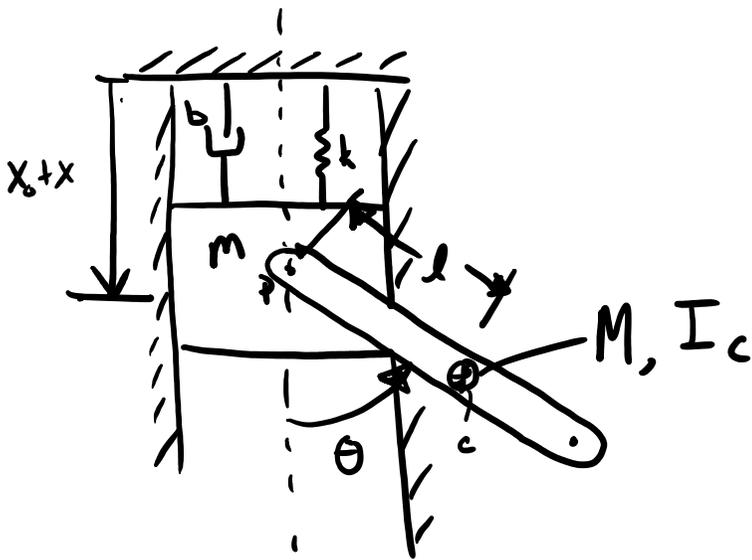
$$\cos \theta \approx 1$$

$$\cos^2 \theta \approx 1$$

$$\sin^2 \theta = \sin \theta \sin \theta \approx 0$$

$$\theta^n \approx 0$$

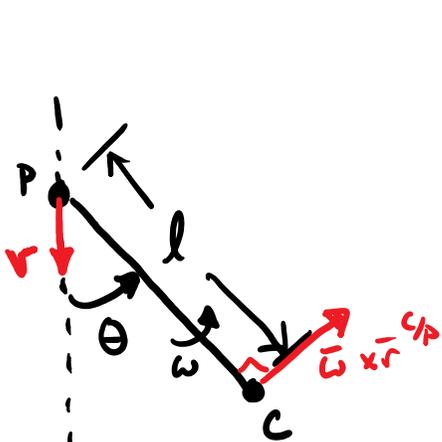
$$\omega^n \approx 0$$



$$\omega = \dot{\theta}$$

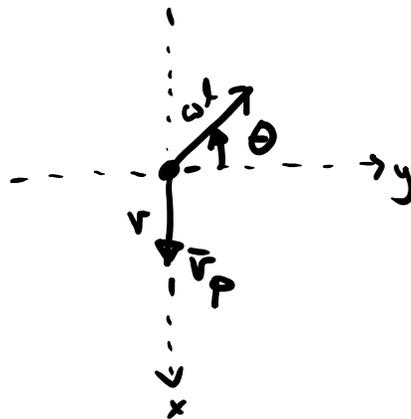
$$v = \dot{x}$$

$$T = \frac{1}{2} m v^2 + \frac{1}{2} M v_c^2 + \frac{1}{2} I_c \omega^2$$



$$\bar{v}_c = \bar{v}_p + \bar{\omega} \times \bar{r}_{c/p}$$

for any two points on a rigid body



$$v^2 = |\bar{v}_c|^2 = |\bar{v}_c^x|^2 + |\bar{v}_c^y|^2$$

$$v^2 = (v - \omega l \sin \theta)^2 + (\omega l \cos \theta)^2$$