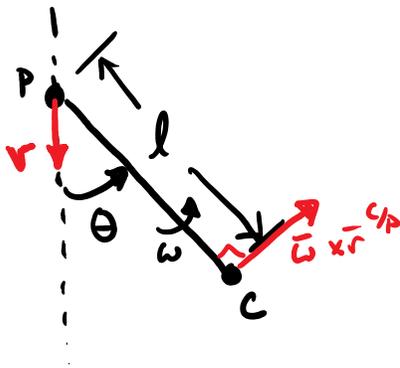


$$\omega = \dot{\theta}$$

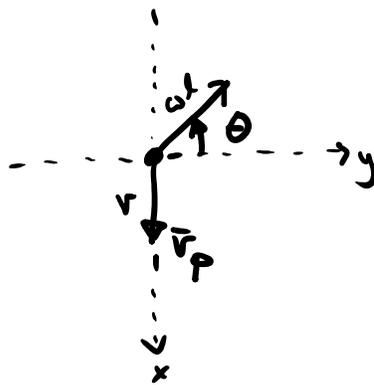
$$v = \dot{x}$$

$$T = \frac{1}{2} m v^2 + \frac{1}{2} M v_c^2 + \frac{1}{2} I_c \omega^2$$



$$\vec{v}_c = \vec{v}_p + \vec{\omega} \times \vec{r}_{c/p}$$

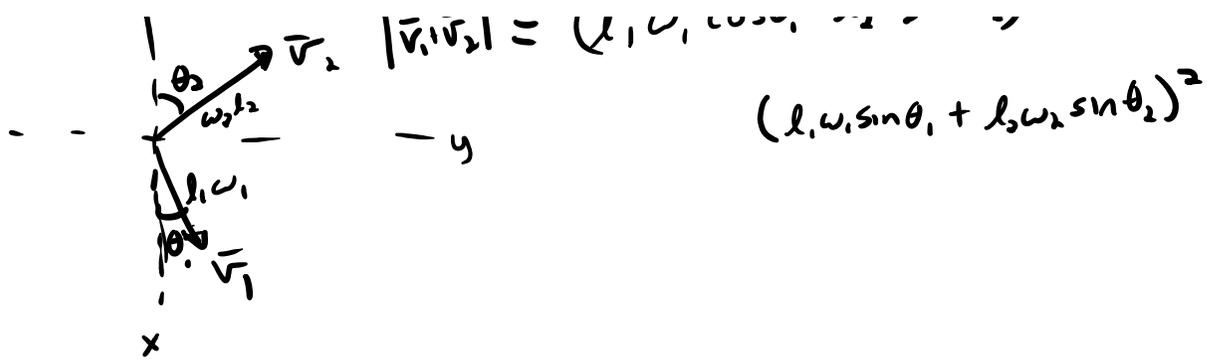
for any two points on a rigid body



$$v_c^2 = |\vec{v}_c|^2 = |\vec{v}_c^x|^2 + |\vec{v}_c^y|^2$$

$$v_c^2 = (v - \omega l \sin \theta)^2 + (\omega l \cos \theta)^2$$

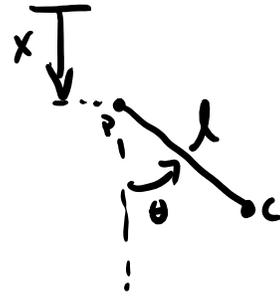
$$|\vec{v}_1 \cdot \vec{v}_2| = (l_1 \omega_1 \cos \theta_1 - l_2 \omega_2 \cos \theta_2)^2 + (l_1 \omega_1 \sin \theta_1 + l_2 \omega_2 \sin \theta_2)^2$$



## Potential energy

$$U = U_{\text{spring}} + U_m + U_M$$

$$U = \frac{1}{2} k x^2 - m g x - M g (x + l \cos \theta)$$



## Q for non-conservative forces

if dealing with viscous damping you can make use of Rayleigh's Dissipation function:

$$R = \frac{1}{2} \sum_{j=1}^N c_j v_j^2$$

for  $N$  viscous damping elements

$v_j \Rightarrow$  magnitude of the velocity

$c_j \Rightarrow$  linear damping coeff for  $v_j$

Then

$$Q_i = - \frac{\partial R}{\partial v_i}$$

$\uparrow$   
 G.S.

for the  $i^{\text{th}}$  Generalized speed

For us:

$$R = \frac{1}{2} b v^2$$

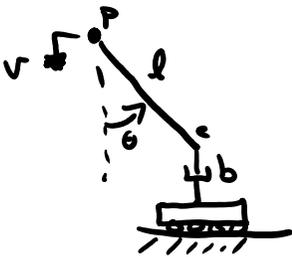
$n$  Lagrange Equations, we have  $n=2$   
 $\uparrow$   
#DoF

$$L = T - U$$

$$\textcircled{1} \quad \dot{v} \quad \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{v}} \right) = \frac{\partial L}{\partial v} + Q_v \quad Q_v = -bv$$

$$\textcircled{2} \quad \dot{\theta} \quad \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}} \right) = \frac{\partial L}{\partial \theta} + Q_\theta \quad Q_\theta = 0$$

What if



$$R = \frac{1}{2} b (v - l \sin \theta)^2 = \frac{b}{2} (v^2 + l^2 \sin^2 \theta - 2vl \sin \theta)$$

$$Q_v = -\frac{\partial R}{\partial v} = -bv + b l \sin \theta$$

$$Q_\theta = -\frac{\partial R}{\partial \theta} = -b \omega l^2 \sin^2 \theta + vl \sin \theta$$

Two EoM

$$\left. \begin{aligned} \textcircled{1} \quad I_{vv} \dot{v} + I_{vv} \ddot{v} + g_v(\omega, v, \theta, x) &= 0 \\ \textcircled{2} \quad I_{\omega\omega} \dot{\omega} + I_{v\omega} \ddot{v} + g_\omega(\omega, v, \theta, x) &= 0 \end{aligned} \right\} \begin{array}{l} \text{two coupled} \\ \text{first order} \\ \text{ODEs} \end{array}$$

$$(3) \quad \dot{\theta} = \omega$$

$$(4) \quad \dot{x} = v$$

$$(1) \&(2) \quad \bar{f} = \overset{I}{\begin{bmatrix} I_{uv} & I_{vw} \\ I_{uw} & I_{vw} \end{bmatrix}} \begin{bmatrix} \dot{s} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} \bar{g} \\ g_w \end{bmatrix} = 0$$

$$\bar{f} = I \dot{s} + \bar{g} = 0$$

$$\boxed{\dot{s} = -I^{-1} \bar{g}} \quad \text{explicit first order form}$$