

- Still working on proposal feedback (one thing: at least a JWF model)
- HW 6 will be due Friday Feb 21 (three problems)
- Modeling report will be due Monday Feb 24

Linearization of Multi-DOF systems

$$\dot{\bar{s}} = \begin{bmatrix} \frac{d}{dt} \left(\frac{\partial L}{\partial v} \right) - \frac{\partial L}{\partial x} - Q_v \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \omega} \right) - \frac{\partial L}{\partial \theta} - Q_\theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

G. coordinate vector
 $\bar{c} = \begin{bmatrix} \theta \\ x \end{bmatrix}$

G. speed vector
 $\bar{s} = \begin{bmatrix} \omega \\ v \end{bmatrix}$

state vector

$$\bar{x} = \begin{bmatrix} \bar{c} \\ \bar{s} \end{bmatrix} = \begin{bmatrix} \theta \\ x \\ \omega \\ v \end{bmatrix}$$

$$\bar{v} = \begin{bmatrix} \bar{c} \\ \dot{\bar{c}} \\ \bar{s} \\ \dot{\bar{s}} \end{bmatrix} = \begin{bmatrix} \theta \\ x \\ \omega \\ v \\ \dot{\theta} \\ \dot{x} \\ \dot{\omega} \\ \dot{v} \end{bmatrix}$$

$$\bar{f}(\bar{c}, \bar{s}, \dot{\bar{s}}) = \bar{f}(\bar{x}, \dot{\bar{x}}) = I \dot{\bar{s}} + \bar{g} = 0 \quad \mathbb{R} \in n \quad n \neq \text{DOF}$$

$$\dot{\bar{c}} = \bar{s}$$

Equilibrium answers: "What are the coordinates when \bar{s} and $\dot{\bar{s}}$ are zero?"

↳ stationary static equilibrium

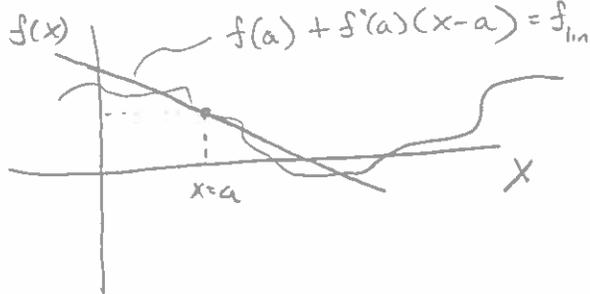
$$\bar{f} \Big|_{\substack{\bar{s} \\ \dot{\bar{s}}}} = 0 \Rightarrow \text{solve for } \bar{c}, \text{ i.e. } \bar{c}_{eq}$$

Now recall a Taylor Series Expansion of scalar function

$f(x)$ about $x=a$:

$$f(x) = \underbrace{f(a) + \frac{f'(a)}{1!}(x-a)}_{\text{linear terms wrt } x} + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots, \quad f' = \frac{df}{dx}$$

linear terms wrt x



The linear approximation is only reasonable at values near $x=a$.

If $f(x)$ is instead a vector function, e.g. $\bar{f}(\bar{c}, \bar{s}, \dot{\bar{s}})$, then the vector form of the first two terms of the Taylor series is:

$$\bar{f}_{lin} = \bar{f}(\bar{v}_{eq}) + J_{f,v}(\bar{v}_{eq}) \cdot (\bar{v} - \bar{v}_{eq}),$$

$$J_{f,v} = \left[\frac{\partial \bar{f}}{\partial \theta}, \frac{\partial \bar{f}}{\partial x}, \frac{\partial \bar{f}}{\partial v}, \frac{\partial \bar{f}}{\partial \omega}, \frac{\partial \bar{f}}{\partial \dot{\omega}}, \frac{\partial \bar{f}}{\partial \dot{v}} \right]$$

$$\text{and } \bar{v}_{eq} = \begin{bmatrix} \bar{c}_{eq} & \bar{s}_{eq} & \dot{\bar{s}}_{eq} \\ \theta_{eq} & \omega_{eq} & 0 & 0 & 0 & 0 \end{bmatrix}$$

The linear equations of motion can be written in a canonical form:

$$\bar{f}_{1,n} = \overset{n \times n}{M} \overset{n \times 1}{\dot{\bar{s}}} + \overset{n \times n}{C} \overset{n \times 1}{\bar{s}} + \overset{n \times n}{K} \overset{n \times 1}{\bar{c}} - \overset{n \times 1}{F} = 0$$

\uparrow mass matrix \uparrow damping matrix \uparrow stiffness matrix \uparrow forcing term (zero if homogeneous)

$$J_{\bar{f}_{1,n}, \dot{\bar{s}}} = M$$

$$J_{\bar{f}_{1,n}, \bar{s}} = C$$

$$J_{\bar{f}_{1,n}, \bar{c}} = K$$

$$\bar{f}_{1,n} \Big|_{\bar{v}=0} = -F$$