

Generalized Forces: non-conservative forces / torques projected onto the "motion space"

all velocities and angular velocities are linear w.r.t the generalized, so we can write any velocity vector like:

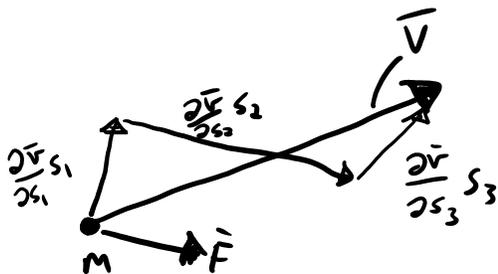
$$\bar{v} = \frac{\partial \bar{v}}{\partial s_1} s_1 + \frac{\partial \bar{v}}{\partial s_2} s_2 + \dots + \frac{\partial \bar{v}}{\partial s_n} s_n + \bar{v}_r$$

$\frac{\partial \bar{v}}{\partial s_i}$ are coefficients of the generalized speeds

and they describe the sensitivity of

\bar{v} to changes in s_j

$\frac{\partial \bar{v}}{\partial s_j}$ are partial velocities (it is a vector)



The generalized forces can be thought of as the component of the force that causes \dot{s}_j of particle m . To find the generalized forces you project the forces acting on the particle onto the partial velocities. The total generalized force

the forces acting on the particle onto the partial velocities. The total generalized force is then:

$$Q_j = \sum_{i=1}^m \bar{F}_i \cdot \frac{\partial \bar{v}_i}{\partial \dot{s}_j} + \sum_{k=1}^p \bar{T}_k \cdot \frac{\partial \bar{\omega}_k}{\partial \dot{s}_j} \quad \text{for } j=1, \dots, n$$

Q_j : generalized force associated with the j^{th} G.S. and the j^{th} Lagrange equation.

n : # of G.S.s (# DoF, # G.C.s)

\bar{F}_i : force applied to the i^{th} point/particle in the system

\bar{v}_i : velocity of the i^{th} point/particle

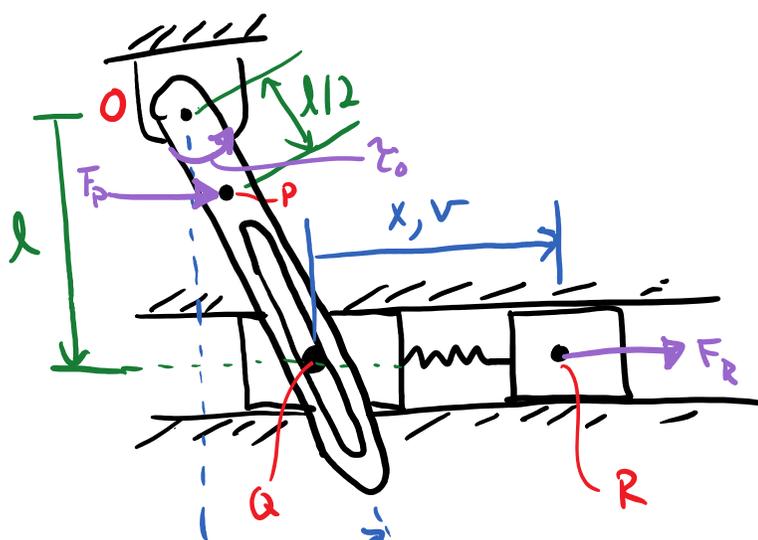
m : # of points/particles with an applied force

\bar{T}_k : torque applied to the k^{th} rotating rigid body

$\bar{\omega}_k$: angular velocity of the k^{th} rotating rigid body

p : # of rigid bodies with an applied torque

Example



Two G.S.: ω, v

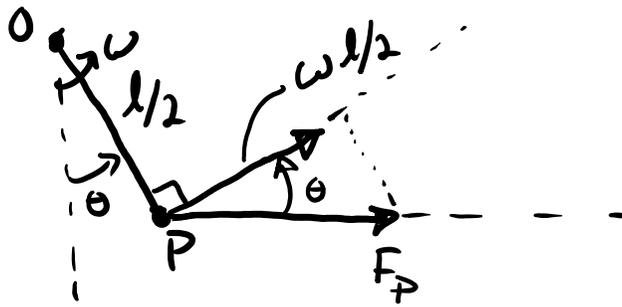
Three forces/torques: γ_0, F_p, \bar{T}_R



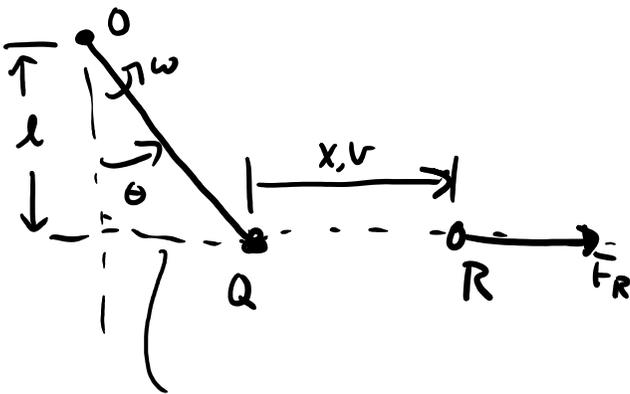
$$Q_\omega = \bar{F}_P \cdot \frac{\partial \bar{v}_P}{\partial \omega} + \bar{F}_R \cdot \frac{\partial \bar{v}_R}{\partial \omega} + \bar{\gamma}_\omega \cdot \frac{\partial \bar{U}}{\partial \omega}$$

$$Q_v = \bar{F}_P \cdot \frac{\partial \bar{v}_P}{\partial v} + \bar{F}_R \cdot \frac{\partial \bar{v}_R}{\partial v} + \bar{\gamma}_\omega \cdot \frac{\partial \bar{U}}{\partial v}$$

Q_ω



$$\begin{aligned} \bar{F}_P \cdot \frac{\partial \bar{v}_P}{\partial \omega} &= F_P \cos \theta \frac{\partial \omega l/2}{\partial \omega} \\ &= F_P l/2 \cos \theta \end{aligned}$$



$$\begin{aligned} \bar{F}_R \cdot \frac{\partial \bar{v}_R}{\partial \omega} &= F_R \frac{\partial (l \omega \sec^2 \theta + v)}{\partial \omega} \\ &= F_R l \sec^2 \theta \end{aligned}$$

$$l \tan \theta \quad \frac{d}{dt} l \tan \theta = l \sec^2 \theta \omega$$

$$\bar{\gamma}_\omega \cdot \frac{\partial \bar{U}}{\partial \omega} = \gamma_\omega \frac{\partial \bar{U}}{\partial \omega} = \gamma_\omega$$

so

$$Q_w = F_P l/2 \cos\theta + F_R l \sec^2\theta + \gamma_0$$

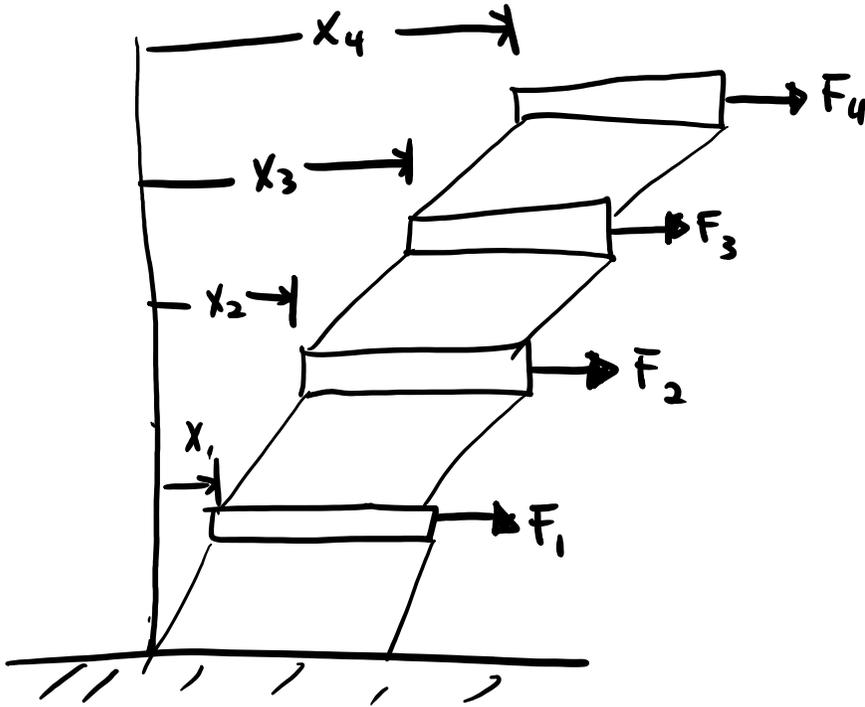
$$\bar{Q}_P = \bar{F}_P \cdot \frac{\partial \bar{U}_P}{\partial v} = F_P \cdot 0 = 0$$

$$\bar{F}_R \cdot \frac{\partial \bar{U}_R}{\partial v} = F_R \cdot 1 = F_R$$

$$\bar{\gamma}_0 \cdot \frac{\partial \bar{U}}{\partial v} = \gamma_0 \cdot 0 = 0$$

$$Q_w = F_R$$

Building under forcing



$$Q_{v_4} = F_4$$

$$Q_{v_3} = F_3$$

$$Q_{v_2} = F_2$$

$$Q_{v_1} = F_1$$

So 4×4 4×1 4×4 4×1 4×1

$$M \ddot{s} + K \dot{c} = \bar{F} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

Investigate $F_{1,2,3,4} = F_0 \sin \omega t$

sinusoidal forcing at various frequencies, ω .